

A Critical System Strength Evaluation of a Power System with High Penetration of Renewable Energy Generations

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Abstract—The power system is experiencing a higher penetration of renewable energy generations (REGs). The short circuit ratio (SCR) and the grid impedance ratio (GIR) are two indices to quantify the system strength of the power system with REGs. In this paper, the critical short circuit ratio (CSCR) is defined as the corresponding SCR when the system voltage is in the critical stable state. Through static voltage stability analysis, the mathematical expression of the CSCR considering the impact of GIR is derived. The maximum value of CSCR is adopted as the critical value to distinguish the weak power system. Based on the static equivalent circuit analysis, it is proved that the CSCR is still effective to evaluate critical system strength considering the interactive impact among REGs. Finally, we find that the GIR can be neglected and the SCR can be used individually to evaluate the system strength when $SCR > 2$ or $GIR > 5$. The correctness and rationality of the CSCR and its critical value are validated on ADPSS.

Index Terms—Critical short circuit ratio, critical value, grid impedance ratio, maximum power transfer theorem, static equivalent circuit analysis.

I. INTRODUCTION

IN recent years, the scale of renewable energy generations (REGs) has increased rapidly. Large-scale REGs are usually located in remote areas and connect to the power system via long-distance transmission lines. The power system is weak when the grid impedance is large. In a weak power system, REGs are easily disconnected due to voltage fluctuations [1]. Even in a relatively strong power system, the grid impedance may significantly change due to faults or operations. Power fluctuations also have adverse impacts on the system voltage stability [2], [3]. The connection of REGs will bring greater challenges to the voltage stability [4]. Therefore, it is significant to accurately evaluate the system strength to promote the acceptance of high penetration of REGs.

The short circuit ratio (SCR) and the grid impedance ratio (GIR) are two indices to evaluate the system strength [5]. Furthermore, the critical short circuit ratio (CSCR) is defined as the corresponding SCR when the system voltage is in the critical stable state. Existing studies mostly focused on the SCR. Many forms of SCR and the critical value to distinguish the weak system were proposed to consider the interactive impact among power electronic equipment. The multi-infeed short circuit ratio [6] and the equivalent effective short circuit ratio [7] were proposed to evaluate the system strength of the multi-infeed HVDC system. The weighted short circuit ratio [8], the composite short circuit ratio [9], and the equivalent circuit-based short circuit ratio [10] were proposed to analyze the voltage stability problem of the weak power system with REGs. The index r [11] was proposed to evaluate the critical system strength based on the static voltage stability limit. The generalized short circuit ratio [12] evaluated the system voltage stability following small disturbances based on the eigenvalue decomposition method. These studies generally assumed that the resistance was much smaller than the reactance in the high-voltage power system, so the impact of resistance was ignored. However, REGs usually connect to the power system via low-voltage transmission lines. The assumption is no longer satisfied. The impact of resistance should not be ignored [13]. Reference [14] studied the impact of GIR on the short circuit quantities of doubly-fed induction generators-based wind turbines. Reference [15] pointed out that the GIR had significant impacts on the CSCR in the low-SCR system by simulations. There is no consensus on the mathematical expression of CSCR. Meanwhile, it is important to analyze the impact of GIR on CSCR.

The rest of this paper is organized as follows: in Section II, the mathematical expression of the CSCR, considering the impact of GIR, is derived for the single-infeed REG system. The maximum value of CSCR is adopted as the critical value to distinguish the weak power system. In Section III, the CSCR of multi-infeed REGs systems is derived. The evaluation effectiveness of the critical value is verified for the power system considering the interaction among REGs. In Section IV, the impact of GIR on CSCR is analyzed. Based on the analysis, the suggestion for the application of SCR and GIR in the system strength evaluation are proposed. In Section V, the validity of the CSCR and critical value are verified on ADPSS. Conclusions are presented in Section VI.

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II. CRITICAL STRENGTH EVALUATION OF SINGLE-INFEED REG SYSTEM

A. Short Circuit Ratio

The system strength is a common concern in a power system with high penetration of REGs. The system strength means the ability of the system to maintain or restore the voltage to the allowable range both during the steady state and following disturbances [16]. The SCR is a common index to quantify the system strength.

The SCR is the ratio of the short circuit capacity at the connection point to the rated capacity of the power electronic equipment connected to that point:

$$\text{SCR} = \frac{S_{\text{ac}}}{P_N} \quad (1)$$

where S_{ac} is the short circuit capacity; P_N is the rated capacity of power electronic equipment, which can be expressed in MW or MVA at the discretion [10].

The short circuit capacity is equal to the short circuit current times the rated voltage [17]. When set, with the rated voltage at the connection point as the reference voltage and the rated capacity of power electronic equipment as the reference capacity, the base impedance Z_b is U_N^2/P_N . Then, SCR can be expressed as:

$$\text{SCR} = \frac{1}{|Z_{\text{p.u.}}|} \quad (2)$$

where U_N is the rated voltage at the connection point; Z is the Thevenin equivalent impedance seen from the connection point; $Z_{\text{p.u.}}$ is the impedance in per unit.

The SCR indicates the ability of the power system to respond to the active/reactive power injection and absorption. The low SCR indicates that the system strength is insufficient to support the reliable operation of power electronic equipment.

B. Grid Impedance Ratio

Based on the Thevenin theorem, the GIR is the ratio of reactance to resistance:

$$\text{GIR} = X/R \quad (3)$$

where X is the equivalent reactance; R is the equivalent resistance; the system impedance Z is $R + jX$.

The GIR can represent the impedance phase angle φ :

$$\varphi = \arctan(X/R) \quad (4)$$

REGs are usually connected to the power system via 35 kV, 110 kV, and 220 kV transmission lines. Typical impedance parameters per kilometer of transmission lines are shown in Table I.

In 35 kV transmission grids, the resistance may be greater than the reactance. Ignoring the impact of resistance will introduce errors to the system strength evaluation. Considering the long transmission distance between the REG and the power system, the system strength evaluation should consider the impact of resistance. The relationship between the resistance and the reactance can be described by the impedance magnitude and the impedance phase angle. According to the definition

TABLE I
TYPICAL IMPEDANCE PARAMETERS PER KILOMETER OF TRANSMISSION LINES

Transmission line type	Rated voltage (kV)	Resistance (Ω/km)	Reactance (Ω/km)	GIR
LGJLJ-35	35	0.910	0.424	0.47
LGJLJ-50	35	0.630	0.412	0.65
LGJ-95	110	0.330	0.416	1.26
LGJ-120	110	0.270	0.409	1.51
LGJ-150	110	0.210	0.403	1.92
LGJ-240	220	0.132	0.342	2.59
LGJQ-300	220	0.107	0.427	3.99
LGJQ-400	220	0.080	0.417	5.21

of SCR and GIR, they correspond to the magnitude and the phase angle respectively.

C. CSCR of Single-infeed REG System

For the HVDC system, the CSCR is defined as the corresponding SCR when the maximum transmittable power point coincides with the rated operating point. The estimated value of CSCR is 2 for the typical inverter design. The CSCR represents the critical strength between the stable and unstable operating region. If the SCR is lower than the CSCR, the operation is in the unstable region [18].

For the power system with REGs, the CSCR is defined as the corresponding SCR when the system voltage is in the critical steady state. The distance between SCR and CSCR is the system stability margin. The P-V curve is usually used to analyze static voltage stability problems [19]. The voltage operating point moves from the upper half to the lower half in the P-V curve with the increase of transmission power. At the inflection point, the system operation turns into the unstable state and the corresponding SCR is the CSCR.

The structure of the single-infeed REG system is shown in Fig. 1. Based on the Thevenin theorem, the AC system is equivalent to a single-power-supply system with a voltage source and an equivalent impedance in series. The REG usually operates with a constant power factor:

$$Q_E = P_E \tan \phi \quad (5)$$

where P_E is the active power; Q_E is the reactive power; ϕ is the power angle.

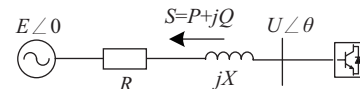


Fig. 1. Structure of the single-infeed REG system.

According to the maximum power transfer theorem, the maximum power injected by the REG and the corresponding limit voltage at the connection point are:

$$P_{\text{max}} = \frac{E^2 \cos \phi}{2|Z|[1 - \cos(\phi - \varphi)]} \quad (6)$$

and

$$U_{\text{max}} = \frac{E}{\sqrt{2}} \cdot \frac{1}{\left(\sqrt{1 - \cos(\phi - \varphi)}\right)} \quad (7)$$

where $P_{\max}$ is the maximum power; $U_{\max}$ is the limit voltage; E is the potential of the AC system voltage source.

Under the circumstances that the power factor of REG and the AC system potential are constant, the maximum power and the limit voltage are related to the system impedance magnitude and the impedance phase angle both.

The complex power injected by the REG at the connection point is:

$$S = P + jQ = (U \cos \theta + jU \sin \theta) \left(\frac{U \cos \theta + jU \sin \theta - E}{R + jX} \right)^* \quad (8)$$

and

$$\begin{cases} P = \frac{U^2 R - UER \cos \theta + UEX \sin \theta}{R^2 + X^2} \\ [5pt] Q = \frac{U^2 X - UEX \cos \theta - UER \sin \theta}{R^2 + X^2} \end{cases} \quad (9)$$

where U is the voltage magnitude; θ is the voltage phase angle; S is the complex power injected by the REG.

Based on the standard trigonometric identity, the quadratic equation in U^2 and the corresponding exact analytical solution expressed in terms of $(U/E)^2$ are:

$$U^4 - [2(PR + QX) + E^2]U^2 + (R^2 + X^2)(P^2 + Q^2) = 0, \quad (10)$$

and

$$(U/E)^2 = \frac{1}{2} \left[1 + 2\lambda \pm \sqrt{1 + 4(\lambda - \mu^2)} \right] \quad (11)$$

where λ and μ are voltage sensitivity indices:

$$\begin{cases} \lambda = \frac{RP + XQ}{E^2} \\ \mu = \frac{XP - RQ}{E^2} \end{cases} \quad (12)$$

The relationship among λ , μ , and SCR can be derived as:

$$|\lambda + j\mu| = \frac{|Z||S|}{E^2} \quad (13)$$

The potential of the AC system voltage source is usually equal to the rated voltage. In order to analyze the maximum transfer capacity of REG, it is considered that the actual output provided by the REG is the rated capacity. The relationship among λ , μ and SCR can be expressed as:

$$|\lambda + j\mu| = \frac{1}{\text{SCR}} \quad (14)$$

And then, λ and μ can be derived from SCR and GIR directly:

$$\begin{cases} \lambda = \frac{1}{\text{SCR}} \cdot \frac{1}{|S|} \cdot \frac{P + Q \cdot (X/R)}{\sqrt{1 + (X/R)^2}} \\ \mu = \frac{1}{\text{SCR}} \cdot \frac{1}{|S|} \cdot \frac{P \cdot (X/R) - Q}{\sqrt{1 + (X/R)^2}} \end{cases} \quad (15)$$

Based on the maximum power transfer theorem, the corresponding system voltage has a uniqueness value at the voltage collapse point. The discriminant equation is equal to 0:

$$1 + 4(\lambda - \mu^2) = 0 \quad (16)$$

Substituting λ and μ into the discriminant equation, the mathematical expression of CSCR can be derived as:

$$\text{CSCR} = \frac{2}{|S|} \left[-\frac{P + Q(X/R)}{\sqrt{1 + (X/R)^2}} + |S| \right] \quad (17)$$

D. Critical Value of Single-infeed REG System

According to the mathematical expression of CSCR, the value range of CSCR can be divided into two scenarios:

1) *GIR tends to zero*

$$\text{CSCR} = 2 - 2P/|S| \quad (18)$$

Due to $0 \leq P \leq |S|$, the value range of CSCR is $[0, 2]$.

2) *GIR tends to infinite*

$$\text{CSCR} = 2 - 2Q/|S| \quad (19)$$

When $0 \leq Q \leq |S|$, the value range of CSCR is $[0, 2]$; when $-|S| \leq Q \leq 0$, the value range of CSCR is $[2, 4]$.

When the GIR is very small, the CSCR is primarily related to the active power injected by the REG. When the GIR is large enough, the CSCR is related to the reactive power flow direction. At present, the REG generally operates with the unity power factor. REGs can provide the reactive power to maintain the voltage stability. It is satisfied when $0 \leq Q \leq P$. So, the value range of CSCR is $[0, 2]$.

Thus, we adopt the maximum value 2 as the critical value to distinguish the weak power system with REGs. That is, when $\text{SCR} > 2$, the power system with REGs can operate in the stable region. When $\text{SCR} \leq 2$, it needs to calculate the CSCR accurately to study whether the power system can maintain the stable operation.

III. CRITICAL STRENGTH EVALUATION OF MULTI-INFEED REGS SYSTEM

A. Analysis Model

It is necessary to build a proper equivalent model for the multi-infeed REGs system. This is necessary because the interactive impact exists not only between the REG and the AC system, but also among REGs. But, the single-port equivalent circuit analysis used for the single-infeed REG system cannot reflect the interaction among REGs. Therefore, the multi-port equivalent circuit analysis is adopted. The structure and the equivalent model of the multi-infeed REGs system are shown in Fig. 2. The equivalent model is referenced in Appendix A for mathematical descriptions.

The system network equation is established based on the admittance matrix:

$$\begin{bmatrix} I_S \\ I_E \end{bmatrix} = \begin{bmatrix} Y_{SS} & Y_{SE} \\ Y_{ES} & Y_{EE} \end{bmatrix} \begin{bmatrix} U_S \\ U_E \end{bmatrix} \quad (20)$$

where U_S is the equivalent potential matrix of the AC system; U_E is the voltage matrix at the connection point; I_S and I_E are the current matrix injected by the AC system equivalent potentials and the REGs respectively; Y_{SS} , Y_{SE} , Y_{ES} , and Y_{EE} are the corresponding sub-matrices of the system admittance matrix.

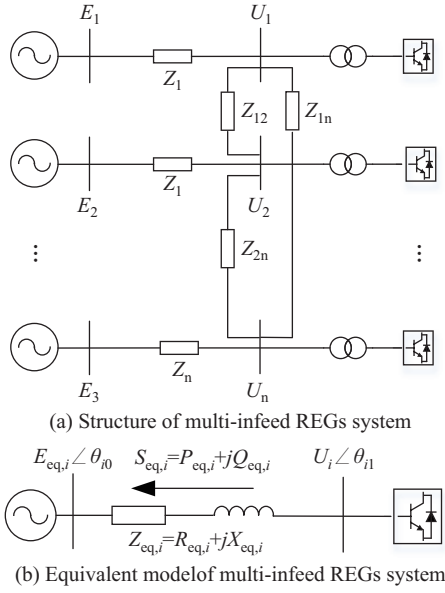


Fig. 2. Diagram of the multi-infeed REGs system.

The i -th element in $\mathbf{U}_E$ is the voltage at the i -th connection point of the system:

$$\mathbf{U}_i = \sum_{j=1}^n \mathbf{A}_{ij} \mathbf{I}_{S,j} - \mathbf{B}_{ii} \sum_{j=1}^n \frac{\mathbf{B}_{ij}}{\mathbf{B}_{ii}} \mathbf{I}_{E,j} \quad (21)$$

and

$$\begin{cases} \mathbf{A} = (\mathbf{Y}_{SE} - \mathbf{Y}_{SS} \mathbf{Y}_{ES}^{-1} \mathbf{Y}_{EE})^{-1} \\ \mathbf{B} = (\mathbf{Y}_{SE} - \mathbf{Y}_{SS} \mathbf{Y}_{ES}^{-1} \mathbf{Y}_{EE})^{-1} \mathbf{Y}_{SS} \mathbf{Y}_{ES}^{-1} \end{cases} \quad (22)$$

where $\mathbf{U}_i$ is the voltage at the i -th connection point; $\mathbf{I}_{S,j}$ and $\mathbf{I}_{E,j}$ are the j -th element in $\mathbf{I}_S$ and $\mathbf{I}_E$ respectively; $\mathbf{A}_{ij}$ is the i -th row and j -th column element of the incidence matrix $\mathbf{A}$; $\mathbf{B}_{ii}$ and $\mathbf{B}_{ij}$ are the i -th diagonal element and the i -th row and j -th column element of the equivalent impedance matrix $\mathbf{B}$.

The mathematical analysis model of the multi-infeed REGs system can be established as:

$$\mathbf{U}_i = \mathbf{E}_{eq,i} - \mathbf{Z}_{eq,i} \mathbf{I}_{eq,i} \quad (23)$$

where, $\mathbf{E}_{eq,i}$ is the corresponding equivalent potential of the AC system; $\mathbf{I}_{eq,i}$ is the equivalent current; $\mathbf{Z}_{eq,i}$ is the equivalent impedance.

The detailed derivation process of the voltage at the i -th connection point is provided in Appendix B. The mathematical expression of electrical quantities in analysis model are provided in Appendix C.

The short circuit capacity and short current according to the definition can be written as:

$$S_{ac,i} = I_{sc,i} \cdot U_{N,i} \quad (24)$$

and

$$I_{sc,i} = \frac{|\mathbf{E}_{eq,i}|}{|\mathbf{Z}_{eq,i}|} \quad (25)$$

where $U_{N,i}$ is the rated voltage at the i -th connection point; $I_{sc,i}$ is the short circuit current at the i -th connection point in the multi-infeed system.

In the maximum power transfer analysis, the equivalent capacity of REG is its equivalent output apparent power. The SCR of the equivalent multi-infeed REGs systems at the i -th point is:

$$SCR_i = \frac{S_{ac,i}}{P_{N,i}} = \frac{|\mathbf{E}_{eq,i}| U_{N,i}}{|\mathbf{Z}_{eq,i}| |S_{eq,i}|} \quad (26)$$

where $S_{eq,i}$ is the equivalent apparent power.

In a single-infeed system, the $\mathbf{E}_{eq,i}$ is equal to $U_{N,i}$. The SCR of the multi-infeed REGs system is consistent with the SCR of the single-infeed REG system.

B. CSCR of Multi-infeed REGs System

Based on the multi-port equivalent circuit analysis, the CSCR of the multi-infeed REGs system has the same form with the CSCR of the single-infeed REG system. The CSCR of the multi-infeed REGs system is:

$$CSCR_i = \frac{2}{|S_{eq,i}|} \left[-\frac{P_{eq,i} + Q_{eq,i} (X_{eq,i}/R_{eq,i})}{\sqrt{1 + (X_{eq,i}/R_{eq,i})^2}} + |S_{eq,i}| \right] \quad (27)$$

C. Critical Value of Multi-infeed REGs System

The value range of CSCR in multi-infeed REGs system also can be divided into two scenarios:

1) The equivalent GIR tends to zero

$$CSCR_i = 2 - 2P_{eq,i}/|S_{eq,i}| \quad (28)$$

Due to $0 \leq P_{eq,i} \leq |S_{eq,i}|$, the value range of $CSCR_i$ is $[0, 2]$.

2) The equivalent GIR tends to infinite

$$CSCR_i = 2 - 2Q_{eq,i}/|S_{eq,i}| \quad (29)$$

When $0 \leq Q_{eq,i} \leq |S_{eq,i}|$, the value range of $CSCR_i$ is $[0, 2]$; when $-|S_{eq,i}| \leq Q_{eq,i} \leq 0$, the value range of $CSCR_i$ is $[0, 2]$.

In a multi-infeed REGs system, when the reactive power flows from the REGs to the power system, the CSCR will not exceed 2.

Based on the maximum power transfer theorem, the static voltage stability can be approximated by the maximum power transmission capacity. Therefore, the static analysis method based on the power flow calculation is also effective to study the static voltage stability problem.

In the multi-infeed REGs system, the complex power at i -th connection point is:

$$S_{eq,i} = \frac{\mathbf{U}_i (\mathbf{U}_i - \mathbf{E}_{eq,i})^*}{\mathbf{Z}_{eq,i}^*} = \frac{1}{\mathbf{Z}_{eq,i}^*} (F_1 - jF_2) \quad (30)$$

and

$$\begin{cases} F_1(|\mathbf{U}_i|, \theta_i) = |\mathbf{U}_i|^2 - |\mathbf{U}_i| |\mathbf{E}_{eq,i}| \cos \theta_i \\ F_2(|\mathbf{U}_i|, \theta_i) = |\mathbf{U}_i| |\mathbf{E}_{eq,i}| \sin \theta_i \end{cases} \quad (31)$$

The corresponding Jacobian matrix is:

$$\mathbf{J} = \begin{bmatrix} 2|\mathbf{U}_i| - |\mathbf{E}_{eq,i}| \cos \theta_i & |\mathbf{U}_i| |\mathbf{E}_{eq,i}| \sin \theta_i \\ |\mathbf{E}_{eq,i}| \sin \theta & |\mathbf{U}_i| |\mathbf{E}_{eq,i}| \cos \theta \end{bmatrix} \quad (32)$$

When the system voltage is in the critical stable state, the Jacobian matrix is singular and its determinant is equal to 0. Then, the equivalent potential and the voltage at i -th connection point meets the following relationships:

$$|E_{eq,i}| = 2|U_i| \cos \theta_i \quad (33)$$

and

$$|U_i|^2 = |Z_{eq,i}| |S_{eq,i}| \quad (34)$$

The CSCR of the multi-infeed REGs system is derived as:

$$CSCR_i = \frac{|E_{eq,i}| |U_{N,i}|}{|Z_{eq,i}| |S_{eq,i}|} = \frac{2U_{N,i} \cos \theta_i}{|U_i|} \quad (35)$$

When the transmission power at the i -th connection point reaches the maximum, the corresponding voltage magnitude is about 0.707 p.u. and the voltage phase angle difference is 45° . Hence, the value of CSCR is:

$$CSCR_i = \frac{2U_{N,i} \cos \theta_i}{|U_i|} \approx 2 \quad (36)$$

The CSCR of the power system with REGs can be unified by the static equivalent circuit analysis both in the single-infeed REG system and the multi-infeed REGs system. The CSCR considers the interaction among REGs. In addition, the critical value derived by the maximum power transfer theorem is consistent with the critical value derived by the static equivalent circuit analysis. This verifies the rationality of the critical value used to evaluate the critical system strength.

D. Consistence of Critical Value of CSCR and Index r

The Jacobian matrix is equal to 0 when the power system voltage is in the critical stable. Based on this [11], proposed an index r and used its critical value 1 to distinguish the weak power system, on condition that the voltage at the connection point is the rated voltage:

$$r = \frac{|U_i|^2}{|Z_{eq,i}| |S_{eq,i}|} = 1 \quad (37)$$

It indicates that the maximum transmissible power is equal to the short circuit capacity at the i -th connection point when the voltage is the rated voltage.

The relationship between the SCR and the index r is:

$$SCR = r \cdot \frac{2U_{N,i} \cos \theta}{|U_i|} \quad (38)$$

When the transmissible power reaches the maximum, the condition does not hold that $|U_i|$ is the rated voltage. The index r should be corrected. The typical value of CSCR based on the index r is:

$$CSCR_i = r \cdot \frac{2U_{N,i} \cos 45^\circ}{0.707U_{N,i}} = 2r = 2 \quad (39)$$

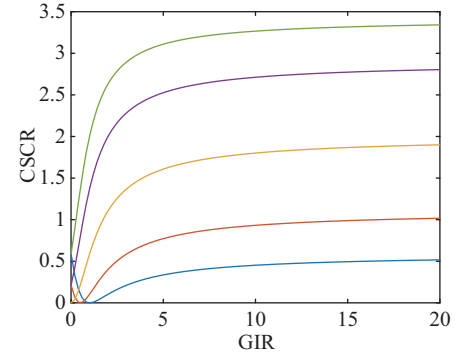
It can be seen that the index r is a special case of the SCR on the circumstance that the voltage at the connection point is the rated voltage. The critical value proposed in this paper is consistent with r . It proves that adopting 2 as the critical value is reasonable.

IV. APPLICATION OF SCR AND GIR IN SYSTEM STRENGTH EVALUATION

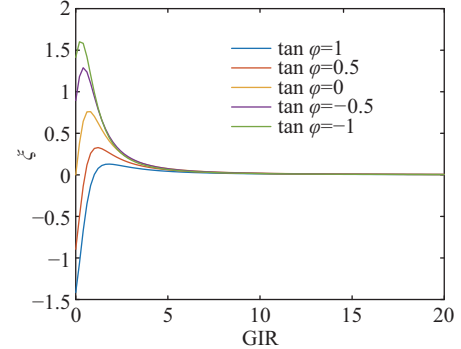
The SCR was a common index used to evaluate the system strength in previous studies. Considering the impact of resistance, the roles of SCR and GIR in the system strength evaluation need to be clarified.

A. Relationship Among CSCR, GIR, and System Strength

The CSCR is the corresponding SCR when the system voltage is in the critical stable state and related to the GIR and the power angle. Several cases with different power factors are taken as examples to show the relationship between CSCR and GIR as shown in Fig. 3(a).



(a) Relationship between CSCR and GIR



(b) Sensitivity of CSCR to GIR

Fig. 3. Relationship and derivation between CSCR and GIR.

If the active power flows from the REG to the power system, the CSCR first decreases and increases subsequently as the GIR increases. It indicates that the system strength first increases and decreases subsequently. If the reactive power flows from the power system to the REG, the CSCR increases as the GIR increases. It indicates that the system strength decreases.

The phenomenon mentioned above can be analyzed by the static voltage stability margin theory. The distance between SCR and CSCR is the stability margin, corresponding to the distance between the system operating point and the maximum transmissible power point. The static voltage stability margin index v_{mg} is:

$$v_{mg} = 1 - \frac{P_0}{P_{max}} \quad (40)$$

where P_0 is the power at the system operating point; P_{max} is the maximum transmissible power.

The sensitivity of v_{mg} to the impedance phase angle is:

$$\rho = \frac{\partial v_{mg}}{\partial \varphi} = \frac{2P_0|Z|\sin(\phi - \varphi)}{E^2 \cos \phi} \quad (41)$$

In the power system with REGs, $\varphi > 0$. The relationship among CSCR, GIR, and system strength is:

1) If $\varphi \in [0, \varphi]$, $GIR \in [0, Q/P]$ and $\rho > 0$.

As the GIR increases, the static voltage stability margin and the system strength increases while the CSCR decreases.

2) If $\varphi = \varphi$, $GIR = Q/P$ and $\rho = 0$.

If the power angle is equal to the impedance phase angle, $CSCR = 0$. The system is always in the stable state.

3) If $\varphi \in [\varphi, \infty]$, $GIR \in [Q/P, \infty]$ and $\rho < 0$.

As the GIR increases, the static voltage stability margin and system strength decreases while the CSCR increases.

B. Impact of GIR on CSCR

Based on the mathematical expression of CSCR, the sensitivity ξ of CSCR to GIR (i.e., X/R) is derived to study the impact of GIR on SCR:

$$\xi = \frac{dCSCR}{d(X/R)} = \frac{2[P(X/R) - Q]}{|S|[1 + (X/R)^2]^{3/2}} \quad (42)$$

The change trend of the impact of GIR on CSCR within different value ranges are shown in Fig. 3(b).

1) When $Q < 0$ and $GIR \in (0, \infty)$, $\xi > 0$ and the CSCR increases monotonically.

2) When $0 < Q$ and $GIR \in (0, Q/P)$, $\xi < 0$ and the CSCR decreases monotonically.

3) When $0 < Q$ and $GIR \in (Q/P, \infty)$, $\xi > 0$ and the CSCR increases monotonically.

The SCR and the static voltage stability margin index both can characterize the system strength. It can be seen that the impact of GIR on CSCR is consistent with the relationship between the impedance phase angle and the static voltage stability margin index.

According to the sensitivity calculation, $\xi < 0.1$ when $GIR > 5$. For the systems with same GIR but different SCR, the GIR has little impact on the CSCR when $GIR > 5$. Under this circumstance, the SCR can be used to evaluate the system strength individually.

C. Evaluation Effectiveness of CSCR when GIR Changes

For the systems with the same SCR but different GIR, the impact of GIR in the system strength evaluation is small and can be ignored when $SCR > 2$. The evaluation effectiveness of SCR when GIR changes can be verified by reversing the CSCR mathematical expression. Within the monotonic interval, the mathematical expression of GIR related with CSCR is:

$$\frac{X}{R} = \frac{PQ + \sqrt{\Delta}}{\Gamma^2 - Q^2} \quad (43)$$

where

$$\begin{cases} \Gamma = \sqrt{P^2 + Q^2} - \frac{S_b}{2} CSCR \\ \Delta = (P^2 + Q^2)\Gamma^2 - \Gamma^4 \end{cases} \quad (44)$$

Considering the actual situation, the CSCR has the following limitations:

1) Δ is always non-negative when $CSCR \in [0, 2]$. In the power system with REGs, the active power flows from the REG to the power system, that is, $P > 0$. At this time, $CSCR \in [0, 2]$.

2) In order to ensure the GIR is meaningful, it is necessary to satisfy $\Gamma^2 - Q^2 > 0$. Meanwhile, $|Q| < |P|$. When $CSCR = 2(|S| - Q)/|S|$, the GIR tends to infinite.

In the reversing process, the original expression needs to satisfy the monotonicity. The curves in different monotonic intervals have different value ranges. According to the mathematical expression of GIR, the changing situation of GIR in monotonic intervals are shown in Fig. 4.

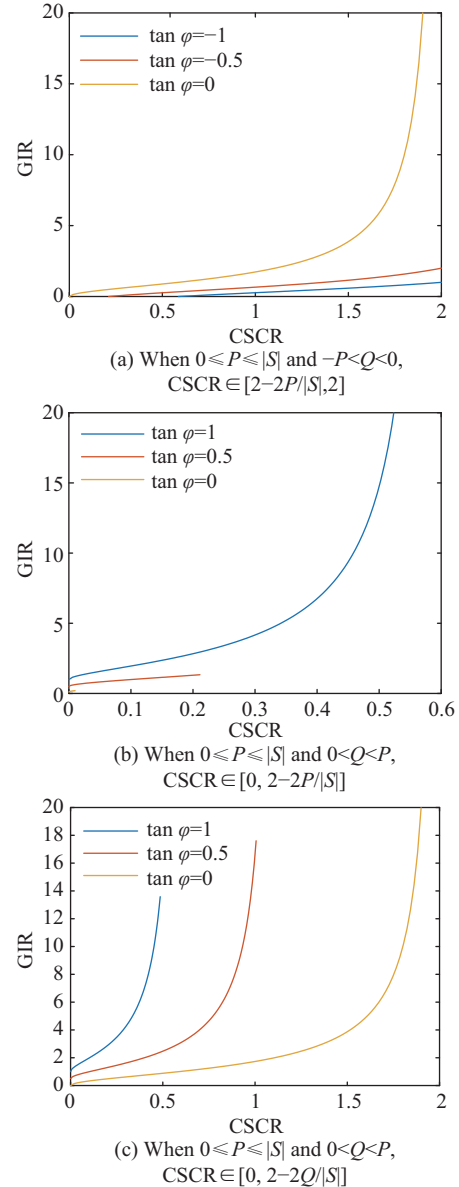


Fig. 4. The changing situation of GIR within monotonic interval.

The changing situation of GIR should be analyzed within one of the following intervals separately:

1) When $0 \leq P \leq |S|$ and $-P < Q < 0$, $CSCR \in [2 - 2P/|S|, 2]$.

2) When $0 \leq P \leq |S|$ and $0 < Q < P$, $CSCR \in [0, 2 -$

$2P/|S|$. At this time, the GIR is relatively small.

3) When $0 \leq P \leq |S|$ and $0 < Q < P$, $CSCR \in [0, 2 - 2Q/|S|]$. At this time, the GIR is relatively large.

Consider the initial condition limits mentioned above, the CSCR does not exceed 2. It means that the system is strong when $SCR > 2$. Under this circumstance, the impact of GIR on the system strength evaluation is small enough to ignore.

D. Application of SCR in System Strength Evaluation

Considering that the REGs connect to the power system via low-voltage transmission grids, it is necessary to consider the impact of resistance. As derived above, the CSCR of the power system with REGs will not exceed 2 and the GIR has little impact on the CSCR when $GIR > 5$. By analyzing the evaluation effectiveness of SCR, it can be seen that the SCR can evaluate the system strength individually when $SCR > 2$ or $GIR > 5$. If the power system neither satisfies $SCR > 2$ nor $GIR < 5$, there will be deviations if you only use the SCR to evaluate the system strength. Under these circumstances, the influence of GIR cannot be ignored. The system strength should be evaluated by exactly calculating the CSCR and using a precise simulation to evaluate the stability.

V. NUMERICAL STUDIES

The correctness of the mathematical expression of CSCR based on GIR derived before and the validity of the critical value to distinguish the weak power system will be researched in this section. ADPSS is an electromagnetic transient simulation tool independently developed by China Electric Power Research Institute and has been widely applied in the State Grid dispatching system. It provides detailed power system operation waveforms in a shorter time scale. Electromagnetic simulation models of the power system with REGs are established for numerical studies. The purpose of numerical studies is to research whether the power systems with different SCR and GIR can operate stably when they are interfaced with the same capacity REGs. Because this occurs, this paper focuses on the strength of the power system, the precondition is made that the control systems of REGs are always stable.

Nowadays, the reactive voltage regulating capacity provided by REGs has not been effectively used, but it is a trend for the future. Therefore, two cases are established. In case A, there is no reactive power exchanges between the REG and the power system. In case B, REGs transmit the reactive power to the power system with a constant power factor. In the two cases, direct-drive wind turbines are adopted to reflect the impacts of REGs. The simulation time is 5 s. Within 0 s to 2 s, the power system builds the stable system voltage. At 2 s, AC system transforms into a Thevenin equivalent system with a certain impedance. Simulation parameters are shown in Table II.

In this section, the relationship between CSCR and GIR is to meet the expression of CSCR. For example, according to the simulation parameters of case A, when $GIR = 1.732$, CSCR is calculated as 1. In case B, when $GIR = 3.71$, CSCR = 0.6.

A. No Reactive Power Injection

According to the mathematical expression of CSCR, the

stability region of the power system with no reactive power transmission is obtained as shown in Fig. 5. When $CSCR = 1$, the system with $GIR = 1.732$ is in the critical stable state; the system with $GIR > 1.732$ is in the unstable state; the system with $GIR < 1.732$ is in the stable state.

TABLE II
SIMULATION SYSTEM PARAMETERS

Symbol	Parameter	Value
U_N	Reference voltage	35 kV
P_N	Reference power	3.6 MW
P_A	Active power of case A	1 p.u.
Q_A	Reactive power of case A	0
P_B	Active power of case B	0.8 p.u.
Q_B	Reactive power of case B	0.4 p.u.

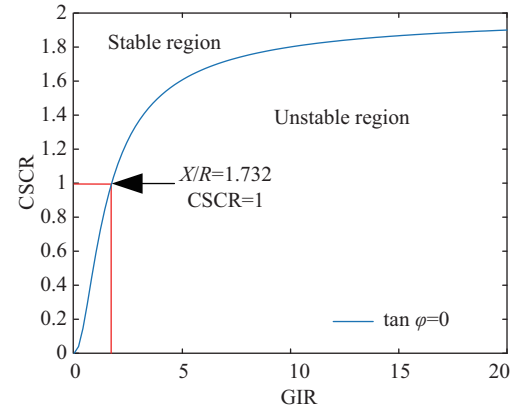


Fig. 5. Stability region of case A.

1) Single-infeed REG System Simulation of Case A

When $CSCR = 1$, the systems with $GIR = 1$, $GIR = 1.6$, $GIR = 2.1$, and $GIR = 5$ are established. When $CSCR = 2$, the systems with $GIR = 0$ and $GIR = \infty$ are established. The system parameters are shown in Table III. The results of the active power injected by the REG are shown in Fig. 6.

TABLE III
SINGLE-INFEED REG SYSTEM PARAMETERS OF CASE A

Number	CSCR	GIR	$R(\Omega)$	$X(\Omega)$
1	1	1.0	240	240
2	1	1.6	180	288
3	1	2.1	146	308
4	1	5.0	67	334
5	2	0	170	0
6	2	∞	0	170

2) Multi-infeed REGs System Simulation of Case A

The simulation structure of the multi-infeed REGs system is shown in Fig. 7. When $CSCR = 1$, the systems with $GIR = 1.6$ and $GIR = 2.6$ are established. When $CSCR = 2$, the systems with $GIR = 0$ and $GIR = \infty$ are established. The system equivalent parameters seen from the 1-th connection point are shown in Table IV. The results of the active power injected by REGs are shown in Fig. 8.

From the simulation results, it can be seen that:

1) For a single-infeed REG system with $CSCR = 1$, when $GIR \leq 1.6$, the system is in the stable state; when $GIR \geq 2.1$, the system is in the unstable state.

TABLE IV
MULTI-INFEED REGS SYSTEM PARAMETERS OF CASE A

Number	CSCR ₁	GIR	$R_{eq}(\Omega)$	$X_{eq}(\Omega)$
1	1	1.60	172	276
2	1	2.60	120	290
3	2	0.16	167	26
4	2	∞	0	165

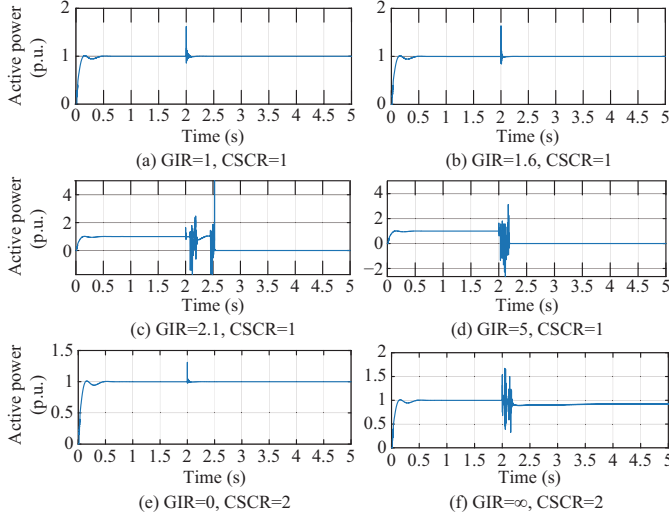


Fig. 6. Simulation results of single-infeed REG systems of case A.

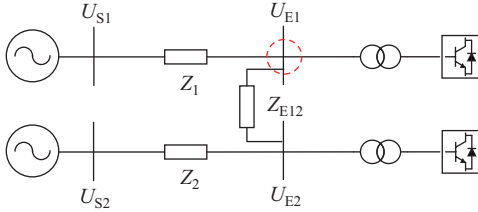


Fig. 7. Simulation structure of multi-infeed REGs system of case A.

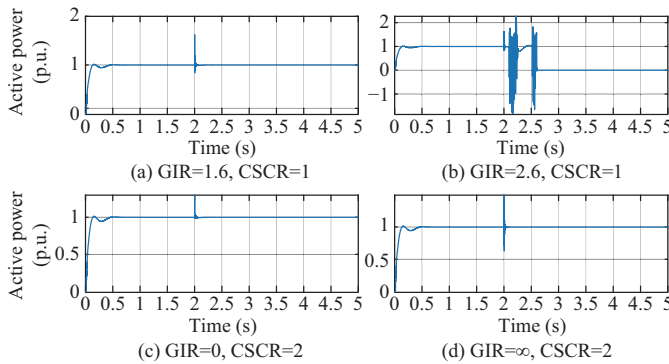


Fig. 8. Simulation results of multi-infeed REGs systems of case A.

2) For a multi-infeed REGs system with $CSCR = 1$, when $GIR \leq 1.6$, the system is in the stable state; when $GIR \geq 2.6$, the system is in the unstable state.

3) the results of 1) and 2) show that when $GIR = 1.732$, $CSCR = 1$ in case A is reasonable.

4) The power system with $SCR = 2$ is always in the stable state, no matter that the value of GIR varies from zero to infinite.

B. Reactive Power Injecting by REGs

According to the mathematical expression of $CSCR$, the stability region of the power system with $\tan \varphi = 0.5$ is shown in Fig. 9. When $CSCR = 0.6$, the system with $GIR = 3.71$ is in the critical stable state; the system with $GIR > 3.71$ is in the unstable state; the system with $GIR < 3.71$ is in the stable state.

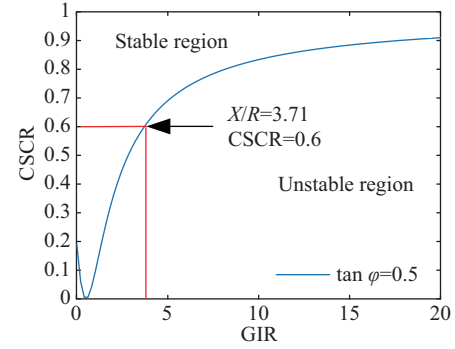


Fig. 9. Stability region of case B.

1) Single-infeed REG System Simulation of Case B

When $CSCR = 0.6$, the systems with $GIR = 3.5$ and $GIR = 4.1$ are established. When $CSCR = 2$, the systems with $GIR = 0$ and $GIR = \infty$ are established. The system parameters are shown in Table V. The simulation results of the active power injecting by REGs are shown in Fig. 10.

TABLE V
SINGLE-INFEED REG SYSTEM PARAMETERS OF CASE B

Number	CSCR	GIR	$R(\Omega)$	$X(\Omega)$
1	0.6	3.5	156	545
2	0.6	4.1	134	551
3	2	0	170	0
4	2	∞	0	170

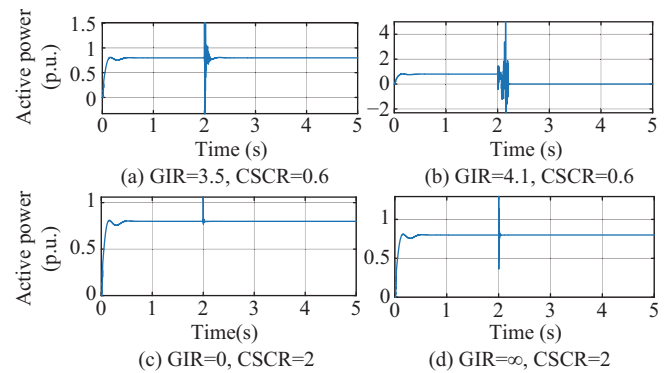


Fig. 10. Simulation results of single-infeed REG systems of case B.

2) Multi-infeed REGs System Simulation of Case B

The simulation structure of the multi-infeed REGs system is the same as Fig. 7. When $CSCR = 0.6$, the systems with $GIR = 3.3$ and $GIR = 4.5$ are established. When $CSCR = 2$, the systems with $GIR = 0$ and $GIR = \infty$ are established. The system equivalent parameters seen from the 1-th connection point are shown in Table VI. The results of the active power injected by REGs are shown in Fig. 11.

TABLE VI
MULTI-INFEED REGS SYSTEM PARAMETERS OF CASE B

Number	CSCR ₁	GIR	$R_{eq}(\Omega)$	$X_{eq}(\Omega)$
1	0.6	3.3	163	536
2	0.6	4.5	123	556
3	2	0.16	167	26
4	2	∞	0	165

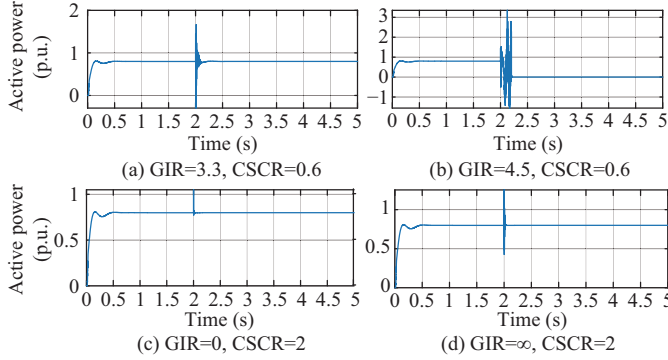


Fig. 11. Simulation results of multi-infeed REGs system of case B.

From the simulation results under the circumstance of $\tan \varphi = 0.5$, it can be seen that:

1) For a single-infeed REG system with CSCR = 0.6, when $GIR \leq 3.5$, the system is in the stable state; when $GIR \geq 4.1$, the system is in the unstable state.

2) For a multi-infeed REGs system with CSCR = 0.6, when $GIR \leq 3.3$, the system is in the stable state; when $GIR \geq 4.5$, the system is in the unstable state.

3) the results of 1) and 2) show that when $GIR = 3.71$, CSCR = 0.6 in case B is reasonable.

4) The power system with $SCR = 2$ is always stable, no matter that the value of GIR varies from zero to infinite.

The simulation results are basically consistent with the mathematical derivations determined by this paper. The correctness of the CSCR and the critical value to distinguish the weak system are verified base on the simulations.

VI. CONCLUSION

In this paper, the mathematical expression of the CSCR is derived considering the impact of the GIR for the power system with high penetration of REGs. We find that the maximum value of CSCR can be used as the critical value to distinguish the weak systems, and its typical value is 2 through a mathematical derivation. Furthermore, when $GIR > 5$ or $SCR > 2$, the impact of the GIR on the SCR can be ignored and the SCR can be individually used to evaluate the system strength. However, when the SCR and the GIR are relatively small ($GIR < 5$ and $SCR < 2$), the impact of GIR should be taken into consideration to avoid deviation when the SCR is used to evaluate the system strength. This is because the impact of the resistance in the impedance is large enough and cannot be neglected in the derivation of the CSCR when the SCR and the GIR are relatively small. Furthermore, the voltage stability is also affected by the control strategies and corresponding dynamic characteristics of REGs, which will be investigated in our future studies.

APPENDIX

A. Equivalent Model of Multi-infeed REGs System

In Fig. 2(b), $E_{eq,i} \angle \theta_{i0}$ is the Thevenin equivalent potential of the AC system seen from the i -th connection point; $Z_{eq,i}$ is the equivalent impedance; $U_i \angle \theta_{i1}$ is the voltage at the i -th connection point; at the i -th connection point, the voltage phase difference θ_i between the equivalent potential and the bus voltage is equal to $\theta_{i1} - \theta_{i0}$; $U_{N,i}$ is the rated voltage at the i -th connection point; $S_{eq,i}$ is the complex power injected by REG into the AC system; the subscript “eq” represents corresponding elements in the equivalent model.

B. Detailed Derivation Process of the Voltage at the i -th Connection Point

Equation (22) can be rewritten as:

$$I_S = Y_{SS}U_S + Y_{SE}U_E \quad (45)$$

and

$$I_E = Y_{ES}U_S + Y_{EE}U_E \quad (46)$$

U_S can be written as:

$$U_S = Y_{ES}^{-1}(I_E - Y_{EE}U_E) \quad (47)$$

By replacing U_S , the expression of U_E is:

$$U_E = AI_S - BI_E \quad (48)$$

where

$$\begin{cases} A = (Y_{SE} - Y_{SS}Y_{ES}^{-1}Y_{EE})^{-1} \\ B = (Y_{SE} - Y_{SS}Y_{ES}^{-1}Y_{EE})^{-1}Y_{SS}Y_{ES}^{-1} \end{cases} \quad (49)$$

The voltage at the i -th connection point U_i is the i -th element of U_E :

$$U_i = \sum_{j=1}^n A_{ij}I_{S,j} - \sum_{j=1}^n B_{ij}I_{E,j} \quad (50)$$

Due to fact that I_S is the current matrix injected by the AC system equivalent potential, the first part in the right can represent the Thevenin equivalent potential of the AC system seen from the i -th connection point as:

$$E_{eq,i} = \sum_{j=1}^n A_{ij}I_{S,j} \quad (51)$$

$Z_{eq,i}$ is the equivalent impedance, which is equal to:

$$Z_{eq,i} = B_{ii} = R_{eq,i} + jX_{eq,i} \quad (52)$$

where B_{ii} is the i -th diagonal element of the equivalent impedance matrix B .

The second part in the right can represent the current at the i -th connection point as:

$$I_{eq,i} = \sum_{j=1}^n \frac{B_{ij}}{B_{ii}} I_{E,j} \quad (53)$$

The mathematical expression of the equivalent system of multi-infeed REGs system can be rewritten as:

$$\begin{aligned} U_i &= \sum_{j=1}^n A_{ij}I_{S,j} - B_{ii} \sum_{j=1}^n \frac{B_{ij}}{B_{ii}} I_{E,j} \\ &= E_{eq,i} - B_{ii}I_{eq,i} \end{aligned} \quad (54)$$

C. Equivalent Parameters of Multi-infeed REGs System

In the derivation of the CSCR of the multi-infeed REGs system, the corresponding equivalent potential $E_{eq,i}$, the current $I_{eq,i}$, the impedance $Z_{eq,i}$, the apparent power $S_{eq,i}$, the active power $P_{eq,i}$, and the reactive power $Q_{eq,i}$, are:

$$E_{eq,i} = \sum_{j=1}^n A_{ij} I_{S,j}, \quad (55)$$

$$I_{eq,i} = \sum_{j=1}^n \frac{B_{ij}}{B_{ii}} I_{E,j}, \quad (56)$$

$$Z_{eq,i} = B_{ii} = R_{eq,i} + jX_{eq,i}, \quad (57)$$

$$S_{eq,i} = E_{eq,i} I_{eq,i}^* = P_{eq,i} + jQ_{eq,i}, \quad (58)$$

and

$$\begin{cases} P_{eq,i} = \frac{C}{R_{eq,i}^2 + X_{eq,i}^2} \\ Q_{eq,i} = \frac{D}{R_{eq,i}^2 + X_{eq,i}^2} \end{cases} \quad (59)$$

where

$$\begin{cases} C = U_i^2 R_{eq,i} - U_i E_{eq,i} R_{eq,i} \cos \theta_i + U_i E_{eq,i} X_{eq,i} \sin \theta_i \\ D = U_i^2 X_{eq,i} - U_i E_{eq,i} X_{eq,i} \cos \theta_i - U_i E_{eq,i} R_{eq,i} \sin \theta_i \end{cases} \quad (60)$$

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